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The exterior unsteady viscous flow and heat transfer due to a porous expanding stretching cylinder

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ABSTRACT

This paper presents a numerical solution of the flow and heat transfer outside a stretching expanding porous cylinder. Under this special boundary condition, the governing system of partial differential equations is converted to a set of coupled ordinary differential equations by using suitable similarity transformations, which are solved by a collocation method equivalent to the fourth order mono-implicit-Runge–Kutta method with MATLAB. The main purpose of the present study is to investigate the effects of the different physical parameters, namely the stretching Reynolds number, the permeability Reynolds number,the expansion ratio and the Prandtl number on the velocity and temperature distribution. The results are shown graphically.

1. Introduction

The studies of laminar flow with permeable walls in an expanding or contracting channel or pipe have received considerable attentions in recent years. The earliest work of unsteady flow in a pipe with expanding or contracting wall can be traced back to Uchida and Ohki [1], who showed that the flow equation can be reduced to a single fourth-order nonlinear ordinary differential equation which included expansion ratio. Goto and Uchida [2] analyzed the incompressible laminar flow in a semi-infinite porous pipe whose radius varied with time. Bujurke et al. [3] obtained a series solution for the unsteady flow in a contracting or expanding pipe. Majdalani et al. [4–6], Dauenhauer and Majdalani [7] obtained both numerical and asymptotical solutions for the different permeability Reynolds number. Recently Asghar et al. [8] discussed the flow in a slowly deforming channel with weak permeability using Adomian decomposition method (AMD). Dinarvand et al. [9,10] obtained analytical approximate solutions for the two dimensional viscous flow through expanding or contracting gaps with permeable walls. Boutros et al. [11,12] discussed the same model in a porous channel or pipe with expanding or contracting walls using Lie group method,respectively. Si et al. [13] also obtained analytical solutions for the asymmetric laminar flow in a porous channel with expanding or contracting walls. Using quasilinearization technique, Srinivasacharya et al. [14] solved numerically the flow of a couple stress fluid in a porous channel with expanding or contracting walls. In seeking further generalization, Xu et al. [15] extended the Dauenhauer–Majdalani model to the case in which the wall expansion ratio α is no longer a constant, but rather a time-dependent variable that varies from α_0 to α_1. As a result, they found the time-dependent solutions to approach the steady state very rapidly. Recently, Si et al. [16–19] extended this model to the viscoelastic and micropolar fluid with the same boundary conditions using homotopy analysis method(-HAM). Si et al. [20–22] discussed the existence of multiple solutions for the flow through porous channel or pipe with expanding or contracting walls using a singular perturbation method. Furthermore, the similarity equation [7] describes the unsteady flow of an incompressible fluid in the expanding porous channel. It is presented by White [23] as one of the new exact Navier–Stokes solutions attributed to Dauenhauer and Majdalani.

However, all the above works considered the flow inside expanding or contracting porous channels and tubes. To the best of our knowledge very little reports were found in literatures for the fluids outside the deforming walls. The pioneering work was done by Wang [24], who considered the flow over stretching cylinders and obtained the asymptotic solution for large Reynolds number using perturbation method. Ishak et al. [25] studied the MHD flow and heat transfer due to a stretching cylinder, and they [26] also investigated the effect of uniform suction/injection on flow and heat transfer due to a stretching cylinder. Both problems are

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solved numerically by the Keller-box method. Aldos and Ali [27] discussed the MHD free forced convection from a horizontal cylinder with suction and blowing. Recently, Fang et al. [28,29] discussed the unsteady viscous flow outside of an expanding or contracting cylinder and there is no permeability on the boundary.

Motivated by the above-mentioned works, The exterior problem is usually difficult to be solved numerically by a direct solver for Navier–Stokes equations because of its unbounded domain. So it is particularly motivated to use a stream function induced transformation, which turns the governing equations into a boundary value problem of a high order ODE. The problem can then be solved by a BVP solver of ODEs. The effects of different parameters, especially the expansion ratio and the Reynolds number,on the velocity fields and temperature distribution are studied and shown graphically.

2. Formulation of the problem

Consider the laminar flow of an incompressible viscous fluid caused by a porous cylinder, whose radius is $a(t)$ and expands or contracts uniformly at a time-dependent rate $\dot{a}(t)$. As shown in Fig.1, the z-axis is measured along the axis of the cylinder and r-axis is measured in the radial direction. The wall has equal permeability v_w. Assume u and v to be the velocity components in the z and r directions, respectively.We also assume that the stretching velocity of the cylinder wall is proportional to the axial distance from the origin,that is, $u = 2kz$. It is assumed that the surface of the cylinder is at a constant temperature T_w and the ambient fluid temperature is T_∞ , where $T_w > T_\infty$. The viscous dissipation is neglected as it is assumed to be small. Under these assumptions, the governing equations are [24–26]:

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0, \qquad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right), \qquad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right), \qquad (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r} = \frac{\kappa}{\rho c}\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2}\right), \qquad (4)$$

where ρ, ν, p, c and κ are density, pressure, kinematic viscosity, specific heat at constant pressure and the coefficient of thermal conductivity, respectively. According to Refs. [11,12,24,29], the boundary conditions are

$$u = 2kz, \quad v = -v_w = -A\dot{a}, \quad T = T_w; \quad r = a(t), \qquad (5)$$

$$v \to 0, \quad T \to 0; \quad r \to \infty, \qquad (6)$$

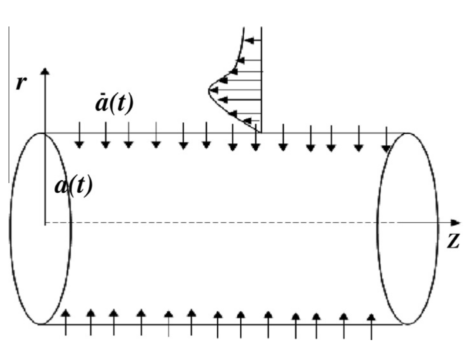

Fig. 1. The porous cylinder with expanding stretching wall.

where $A = \frac{v_w}{\dot{a}}$ is the measure of wall permeability [11,12] and k is a constant representing the stretching strength [29].

Introduce the stream function [12]

$$\psi = \nu z F(\xi, t), \qquad (7)$$

where $\xi = \frac{r}{a}$. The velocity and the temperature can be written as

$$u = \frac{\nu z F_\xi(\xi, t)}{a^2 \xi} \quad \text{and} \quad v = -\frac{\nu F(\xi, t)}{a\xi}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}. \qquad (8)$$

Substituting ψ, θ into Eqs. (1)–(4), one obtains differential equations. These are

$$\frac{F_{\xi\xi\xi}}{\xi} + \frac{F_{\xi\xi}}{\xi}\left(\frac{F}{\xi} - \frac{1}{\xi} + \alpha\xi\right) - \frac{F_\xi}{\xi}\left(\frac{F_\xi}{\xi} + \frac{F}{\xi^2} - \frac{1}{\xi^2} - \alpha\right) - \frac{a^2}{\nu}\frac{F_{\xi t}}{\xi} = 0, \qquad (9)$$

$$\theta_{\xi\xi} + Pr\left(\frac{F}{\xi}\theta_\xi + \theta_\xi \xi\alpha - a^2\nu^{-1}\theta_t\right) = 0, \qquad (10)$$

where $Pr = \frac{\rho c \nu}{\kappa}$ is the Prandtl number and $\alpha = \frac{a\dot{a}}{\nu}$ is the wall expansion ratio. Note that the expansion ratio will be positive for expansion and negative for contraction.

A similar solution with respect to both space and time can be developed following the transformation described by Uchida and Aoki [1], Majdalani and Zhou [4] and Dauenhauer and Majdalani [7], respectively. This can be accomplished by considering in the case: α is a constant and $F = F(\xi)$, it leads to $F_{\xi t} = 0$. Here, we assume that $\theta_t = 0$. From a physical standpoint [1,4,7], our idealization is based on a decelerating expansion rate that follows a plausible model,

$$\alpha = \frac{\dot{a}a}{\nu} = \frac{\dot{a}_0 a_0}{\nu} = \text{constant}, \qquad (11)$$

where a_0 and $\dot{a}_0$ denote the initial pipe radius and expansion rate, respectively. As a result, the rate of expansion decreases as the internal radius increases. Integrating Eq. (11) with respect to time, the similar solution can be achieved. The result is

$$\frac{a}{a_0} = \frac{v_w(0)}{v_w(t)} = \sqrt{1 + 2\nu\alpha t a_0^{-2}}. \qquad (12)$$

Since $v_w = A\dot{a}$ and $A = \text{constant}$ [4,7,11,12], then the expression for the injection velocity also can be determined. Under these assumptions, Eqs. (9) and (10) become

$$\frac{F_{\xi\xi\xi}}{\xi} + \frac{F_{\xi\xi}}{\xi}\left(\frac{F}{\xi} - \frac{1}{\xi} + \alpha\xi\right) - \frac{F_\xi}{\xi}\left(\frac{F_\xi}{\xi} + \frac{F}{\xi^2} - \frac{1}{\xi^2} - \alpha\right) = 0, \qquad (13)$$

$$\theta_{\xi\xi} + Pr\left(\frac{F}{\xi}\theta_\xi + \theta_\xi \xi\alpha\right) = 0. \qquad (14)$$

Introduce a new transformation

$$\eta = \xi^2, \qquad (15)$$

then Eqs. (13) and (14) become

$$\eta F''' + F'' + \frac{1}{2}(F''F - F'^2) + \frac{\alpha}{2}(\eta F'' + F') = 0. \qquad (16)$$

$$\theta''\eta + \frac{\theta'}{2}(Pr\eta\alpha + PrF + 1) = 0, \qquad (17)$$

The boundary conditions are translated into

$$F'(1) = Re^*, \quad F(1) = Re, \quad F'(\infty) = 0, \qquad (18)$$

$$\theta(1) = 1, \quad \theta(\infty) = 0, \qquad (19)$$

where $Re^* = \frac{ka^2}{\nu}$ is the Reynolds number for the wall stretching and $Re = \frac{a v_w}{\nu}$ is the permeability Reynolds number. Note that Re is positive for injection and negative for suction.

Let

$$f = \frac{F}{Re}, \qquad (20)$$

Eqs. (16) and (17) become

$$\eta f''' + f'' + \frac{Re}{2}(f''f - f'^2) + \frac{\alpha}{2}(\eta f'' + f') = 0. \tag{21}$$

$$\theta''\eta + \frac{\theta'}{2}(Pr\eta\alpha + PrRef + 1) = 0, \tag{22}$$

The corresponding boundary conditions (18) and (19) become

$$f'(1) = s, \quad f(1) = 1, \quad f'(\infty) = 0, \tag{23}$$

$$\theta(1) = 1, \quad \theta(\infty) = 0. \tag{24}$$

where $s = \frac{Re^*}{Re}$.

3. Results and discussion

It is difficult to obtain directly the analytical solution of the coupled ordinary equations. Eqs. (21) and (22) together with the boundary conditions (23) and (24) are obtained using Bvp4c in MATLAB [30]. In general, Bvp4c is to replace the boundary condition at infinity with one at finite point. In order to obtain different branches of the solution, we need to adjust the initial mesh,the suitable guess and the length of the interval for the solution. This idea has been discussed in the Ref. [22] in detail and a problem with multiple solutions has been solved successfully. The infinite of η equals 15 or 25 and the maximum residual of the solution is no more than 10^{-4}. The effects of the governing parameters, such as the expansion ratio α, the Reynolds number Re and the parameter s, on the flow are investigated. During the numerical calculation, dual solutions are found for an expanding cylinder.

Figs. 2 and 3 illustrate the two branches of the shear stress $f''(1)$ and the $\theta'(1)$ as a function of the Reynolds number Re and the parameter s as other parameters are fixed. Both figures show the existence of dual solutions for particular values of physical parameters. In the following discussion, the first and second solutions are classified as the shape of axial velocity. It can be seen that $Re = 0$ is the discontinuity point of $f''(1)$ and $\theta'(1)$. Moreover, the solutions exist up to the critical values of Re, say Re_{key}, which are different with different s. For example, as $s = 0.2$, $Pr = 0.5$, $\alpha = 5$, the $Re_{key} = -4.718$, and as $s = 0.5$, $Pr = 0.5$, $\alpha = 5$, $Re_{key} = -3.2635$. There are two solutions as $Re > Re_{key}$, and no solution when $Re < Re_{key}$. Furthermore, the value of $f''(1)$ tend to ∞ as $Re \to 0$ for the second solution. In the following Figures, only the second solution is labeled, and then the one without being labeled is the first solution.

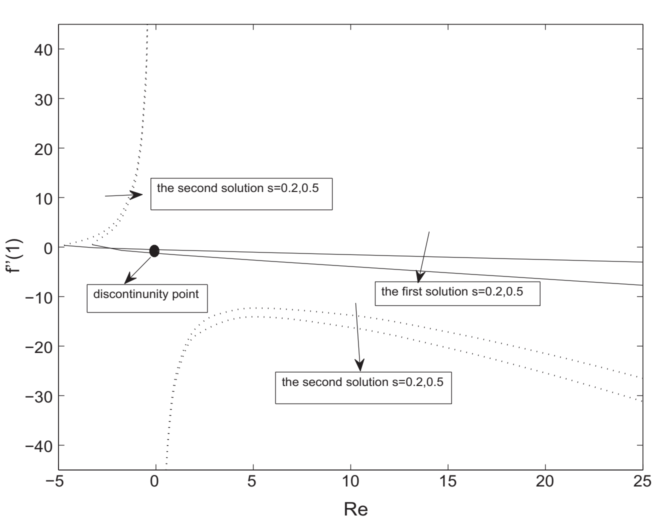

Fig. 2. The two solutions of $f''(1)$ for different s as $\alpha = 5$, $Pr = 0.5$.

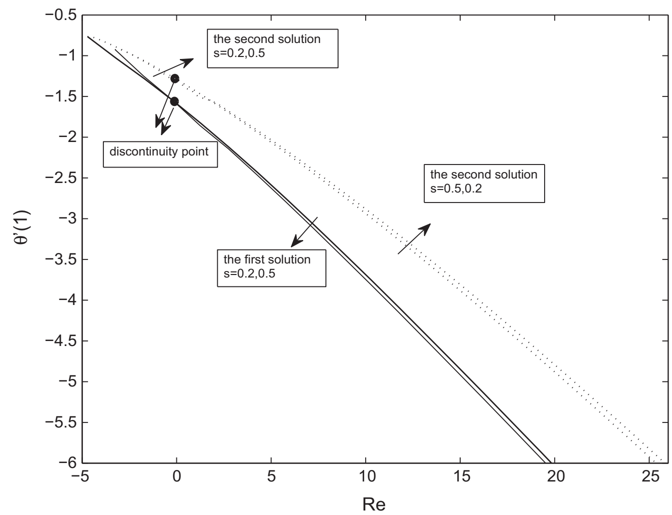

Fig. 3. The two solutions of $\theta'(1)$ for different s as $\alpha = 5$, $Pr = 0.5$.

Fig. 4 shows the two solutions influenced by the expansion ratio when Re is positive or negative, respectively. It can be seen that the first solution is similar no matter the Reynolds number Re is positive or not, which both are the decreasing function of expansion ratio α. However for the second solution, the influence of Reynolds number Re is different. As Reynolds number is positive, $f'(\eta)$ first decreases to a local minimum velocity and then increases back to zero as η increases to infinity. Furthermore, the thickness of the boundary layer decreases with the increase of expansion ratio α, which in another point shows that the parameter α influences the form of the boundary layer. However, as Reynolds number is negative, the velocity $f'(\eta)$ will reach at a maximum point and then vanish gradually to zero as η increases.

Fig. 5 illustrates the effects of s on the velocity and temperature distribution. We can find that the profiles of velocity is different dramatically compared with the above figures. The axial velocity at the wall is different since there are different stretching Reynolds number Re^* at the wall of the tube, then $f'(\eta)$ on the wall is not zero. For the first solution, the axial velocity is an increasing function of s and all decrease to zero as $\eta \to \infty$. However, for the temperature, it is a decreasing function and the influence of different s is little. The axial velocity is an increasing function of s, which becomes zero at large distance from the surface of the cylinder. For the second solution, the axial velocity decreases from different value near the wall to the local minimum point and then increases to zero as η tends to infinity.

Fig. 6 shows the effects of Reynolds number on the velocity as the parameter s and expansion ratio are fixed. For the second solution,when there exists suction velocity on the wall, the axial velocity increases when the cylinder is expanding. When $\eta \to \infty$, the axial velocity also becomes zero. The magnitude of the peak of the axial velocity also increases with the increasing magnitude of the Reynolds number when the cylinder is expanding. While for the first solution, the velocity is the increasing function of Reynolds number Re. We can find that the influence of the Reynolds number on the temperature is obvious. The temperature is a decreasing function of Reynolds number Re, and finally vanishes at some large distance from the surface of the cylinder.

Fig. 7 exhibits the velocity and temperature profiles for various values of the positive Reynolds number Re as the expansion ratio and the parameter s are fixed. For the second solution, the axial velocity is a increasing function of Re. However, the influence on the first solution of the axial velocity is very little.

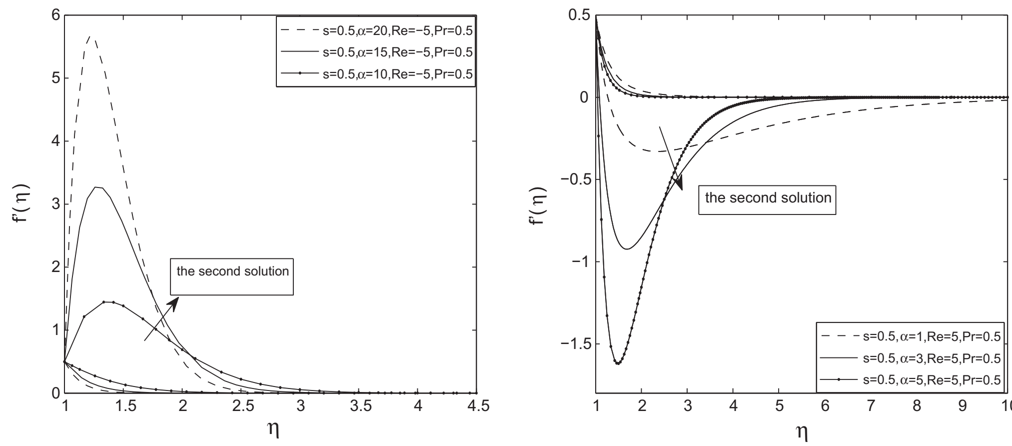

Fig. 4. Variation of $f'(\eta)$ for different α as $Re = 5$ and $Re = -5$, respectively.

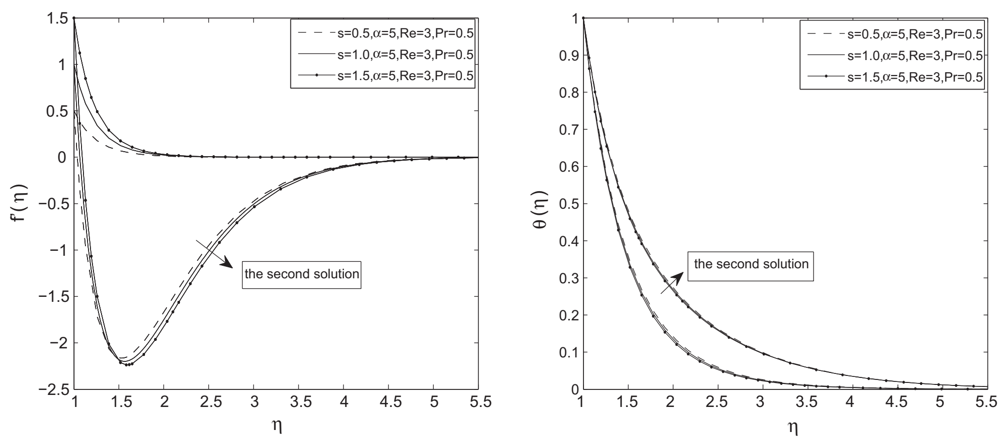

Fig. 5. Variation of $f'(\eta)$, $\theta(\eta)$ for different s.

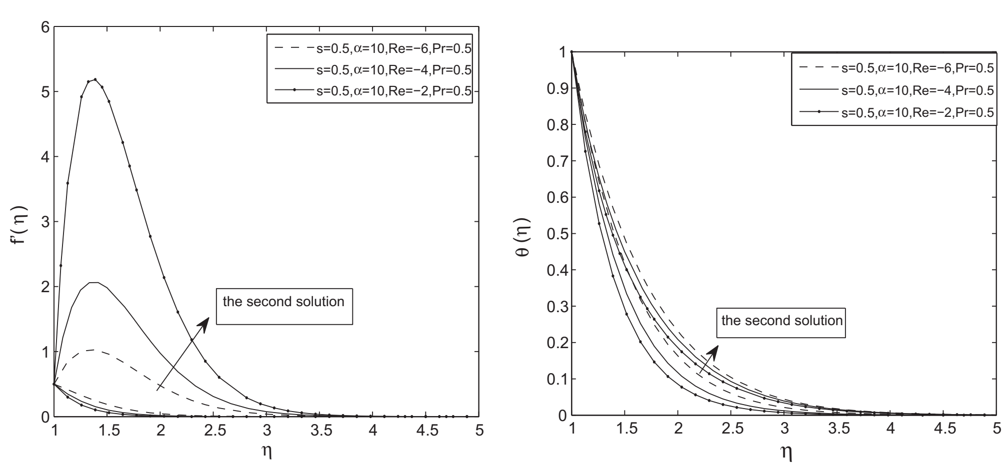

Fig. 6. Variation of $f'(\eta)$, $\theta(\eta)$ for different negative Re.

When we consider the two solution of the temperature. Both temperature distributions are also found to decrease as Re increases, and finally vanishes at some large distance from the surface of the cylinder.

4. Conclusion

In summary, the numerical solution for the laminar flow and heat transfer outside a porous deforming pipe is obtained by a

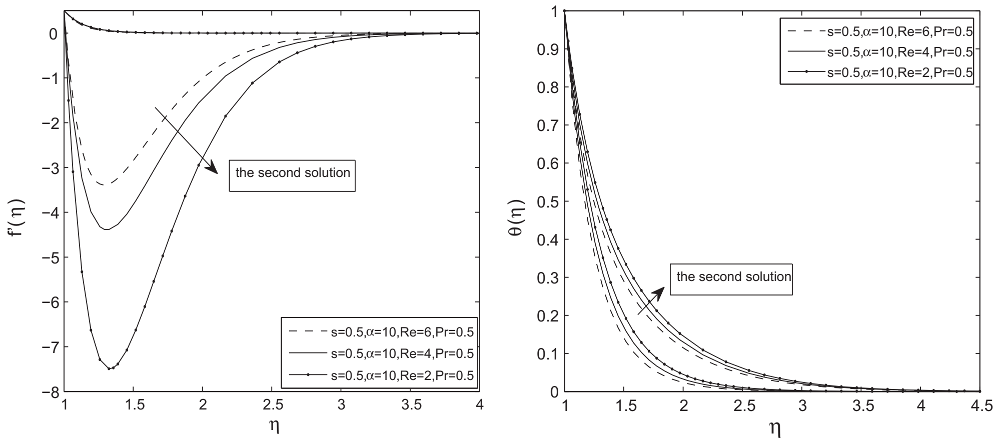

Fig. 7. Variation of $f'(\eta)$, $\theta(\eta)$ for different positive *Re*.

combined analytical/numerical method. Using suitable transformations, the Navier–Stokes equations and the energy equation are reduced to a coupled nonlinear ordinary differential equations. A important conclusion can be drawn that two solutions exist for different parameter *s*. Furthermore, For the second solution, the influence of Reynolds number is different. when the Reynolds number is negative, the axial velocity first increase at a maximum point and then decrease to zero. However, when the Reynolds number is positive, the trend of the axial velocity is inverse. For both cases, the axial velocity vanishes at zero as η tends to infinity.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.compfluid.2014.09.038.

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